Analysis of TU Dresden IAC-21 test results.

This note presents an analysis of the microwave and superconducting test results which were presented in the TU Dresden IAC-21 paper entitled, *Thrust Measurements of Microwave, Superconducting and Laser Type EmDrives.*

1.Theory

The theory of microwave EmDrive operation requires an electromagnetic wave to be propagated along the axis of a waveguide, with both ends closed by end plates. With the correct dimensions, frequency and mode of transmission, the waveguide forms a resonant cavity. At one end plate, the guide velocity of the EM wave must be significantly less than the guide velocity at the other end. This can be achieved by tapering the waveguide or by inserting a dielectric at one end. The difference in guide velocities gives rise to a difference in reflection force, so called radiation pressure, at each end plate.

This difference in end plate forces results in the cavity producing thrust, which, if it is free to move, will cause the cavity to accelerate according to Newton's laws. The momentum acquired by the cavity will be equal and opposite to the momentum lost by the EM waves over the multiple reflections. Thus EmDrive complies with the law of Conservation of Momentum. The Kinetic energy gained by the accelerating cavity will cause a partial loss of stored energy in the resonant cavity and thus a reduction of thrust. This demonstrates compliance with the law of Conservation of Energy. Maximum thrust can be measured by restricting acceleration in a correctly designed balance apparatus, but if there is no acceleration, the thrust will be counteracted by a Reaction force.

As the theory is completely based on classic physics, any test cavity must be designed using classic microwave engineering equations which are available in any good text book. However to help the understanding of the following analysis, standard waveguide transmission equations are reproduced here.

The equation references are from:

P.A.Rizzi, Microwave Engineering Passive Circuits, Prentice-Hall, New Jersey 1988.

Free space wavelength $\lambda_0 = \frac{c}{f}$ [2-54]

Guide wavelength $\lambda_g = \frac{\lambda 0}{\sqrt{\mu R \mathcal{E} R - \left\{\frac{\lambda 0}{\lambda c}\right\}^2}}$ [5-70]

Cut off wavelength for circular waveguide $\lambda_c = K_{nm}D$ [5-69]

Cut off frequency $F_c = \frac{c}{\lambda c \sqrt{\mu R \mathcal{E} R}}$ [5-68]

The values for K_{nm} are given in the tables from the reference text book [Table 5-3]

m n	1	2
0	0.82	0.448
1	1.706	0.589
2	1.029	0.468

Knm for TE modes

m n	1	2
0	1.306	0.569
1	0.820	0.448
2	0.612	0.373

Knm for TM modes

2. Mode Analysis of Superconducting Cavity

The equations were applied, in a mode analysis, to the tests for the superconducting cavity, which were given in figs 26 and 27 of the paper. The dimensions were given in fig 7 of the paper. The mode analysis results are shown in the table below.

Note. p is the number of half wavelengths, which must be an integer for resonance, and the test frequency must be above F_c to avoid cut off.

It is assumed that the dielectric is not matched to the wave impedance of the cavity as the diameter is so close to the internal diameter of the cavity. The relative permittivity for the dielectric material is not quoted, but the mismatch is estimated to be around 3:1 for HDPE at frequencies close to 2GHz. Therefore the interface will simply reflect the EM wave, and p is calculated for the vacuum section only.

Freq (MHz)	λ_0 (mm)	Mode	Knm	λ_c (mm)	λ_g (mm)	F_c (MHz)	p	Notes
1,843	163	TE01	0.82	131.2	∞	2,285	0	+dielectric. Cut off
1,843	163	TE11	1.706	272.96	202.57	1,098	1.678	+dielectric. Not resonant
1,843	163	TE12	1.029	164.64	1.054.6	1,821	0.322	+dielectric. Not resonant
1,843	163	TM11	1.306	208.96	259.16	1,435	1.312	+dielectric. Not resonant
1,819	165	TE01	0.82	131.2	∞	2,285	0	Cut off
1,819	165	TE11	1.706	272.96	206.76	1,098	2.031	Resonant TE112
1,819	165	TE12	1.029	164.64	∞	1,821	0	Cut off
1,819	165	TM11	1.306	208.96	268.11	1,435	1.566	Not resonant
1,959	153	TE01	0.82	131.2	∞	2,285	0	Cut off
1,959	153	TE11	1.706	272.96	184.82	1,098	2.273	Not resonant
1,959	153	TE12	1.029	164.64	415.01	1,821	1.012	Resonant TE121
1,959	153	TM11	1.306	208.96	224.76	1,435	1.869	Not resonant

Results of Mode Analysis for superconducting cavity tests.

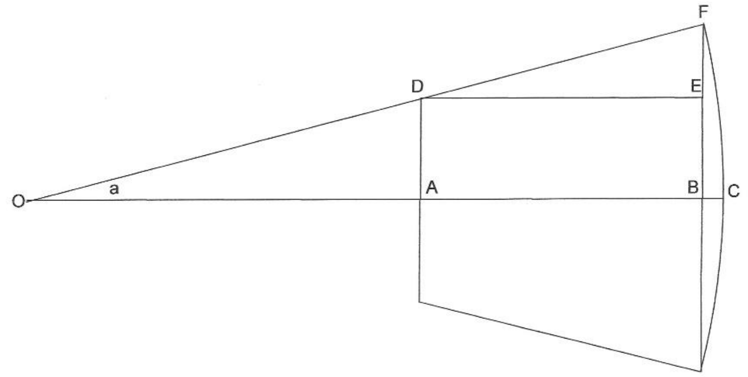
The only resonant conditions are at 1,119MHz and 1,959MHz without the dielectric, whilst the 1,843MHz which is present in both configurations is likely to be the resonant frequency of the loop itself. This suggests that the loop impedance is also not matched to the cavity which would explain the very low Q values of 2,600 at 1,819MHz and 2,063 at 1,959MHz.

Clearly with no dielectric, which are the only circular waveguide tests to show cavity resonance, no difference in guide velocity is present, and therefore no thrust will be generated.

3. Geometry of Microwave Tapered Cavity

One essential requirement for a resonant cavity is that the path length between the end plates is identical for all points across the wave-front. Any errors in path length will cause phase distortion across the wave-front, which will quickly build up with the multiple paths in a resonant cavity. Phase errors will also cause the angle formed between any EM path and the end plates to deviate from the orthogonal. Thus sidewall reflection will take place which will negate the net force at the end plates.

Applying simple trigonometry to the tapered cavity design, with a flat small end plate and a spherical large end plate, as shown in fig.3 of the paper.



EF = $\frac{279.4}{2} - \frac{158.8}{2}$ = 60.3mm

Tan a = $\frac{EF}{DE}$ Then Tan a = $\frac{60.3}{228.6}$ Then a = 14.7769 deg

Sin a = $\frac{EF}{DF}$ Then DF =236.4 **i.e.Path length along side-wall =236.4mm.**

Also Sin a = $\frac{AD}{0D}$

Then OD = 311.2mm

and OF =547.6mm. Thus radius of spherical end plate is 547.6mm.

Tan a = $\frac{AD}{OA}$ Then OA = 301mm

AC =547.6 – OA. Then AC = 246.6mm **i.e. Path length along axis = 246.6mm.**

Then path length difference for one EM transit = 246.6 – 236.4 = 10.2mm

Assume p=2 and one wavelength = AC

Then phase error across wave-front for one EM path = $\frac{10.2}{246.6}$ π = 14.9 degrees.

Now resonance completely disappears when the phase error reaches 180 degrees.

Therefore the maximum number of single EM paths is $\frac{180}{14.9}$ = 12.1

Note that the Q of a cavity can be approximated to the number of return paths travelled by the EM wave. Thus a maximum Q would be around 6, which does not correspond to any Q values given in Table 1 of the paper. These Q measurements were simply resonances of the input loop due to a variety of spurious modes. These can always be seen when a high Q loop is energised in any shape of cavity. They are not the measurement of axial resonance between the end plates.

The microwave tapered cavity will clearly be incapable of resonating along the axis of the taper and no thrust will be generated.

Conclusions.

From the above analyses, it can be seen that no thrust could be expected from any of the tests carried out on the two microwave cavities, described in the paper. The analyses were based on classic physics, simple trigonometry and long established principles of microwave engineering.

Future work could include correcting the geometry of the tapered cavity with a spherical small end plate. The electrical length of the cavity can be calculated by integrating the incremental guide wavelength for the resonant frequency and mode. The design resonant frequency can then be compared with the measured value, and the loop impedance varied until the correct match with the cavity impedance is achieved.

For a superconducting thruster, the existing substrates could be incorporated into the tapered cavity, with the large end plate modified for a flat central section and a spherical annular section. This is described in reference [6] of the paper where it is illustrated in Fig.4b.