IAC- 08 – C4.4.7

MICROWAVE PROPULSION – PROGRESS IN THE EMDRIVE PROGRAMME

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ABSTRACT

This paper provides an update on the programme of research and development into a new form of electric propulsion. EmDrive technology provides direct conversion of electrical energy to thrust, using radiation pressure at microwave frequencies in a tapered, high Q, resonant cavity. The work has been carried out with both UK government funding and private investment.

A summary of the theory behind EmDrive is given, followed by answers to the most frequently asked questions concerning the production of net force, conservation of momentum and conservation of energy. The theory clearly derives equations for both static and dynamic thrust from the basic laws of physics.

The experimental work completed to date is described. The paper also gives a progress report on current programmes. These include an experimental superconducting thruster and development of a flight thruster.

A Demonstrator microsatellite proposal is also described, propelled by the flight thruster.

1. BASIC PRINCIPLES

The technique described in this paper uses radiation pressure, at microwave frequencies, in an engine which provides direct conversion from microwave energy to thrust, without the need for propellant.

The concept of the microwave engine is illustrated in fig 1. Microwave energy is fed from a magnetron, via a tuned feed to a closed, tapered waveguide, whose overall electrical length gives resonance at the operating frequency of the magnetron.

The group velocity of the electromagnetic wave at the end plate of the larger section is higher than the group velocity at the end plate of the smaller section. Thus the radiation pressure at the larger end plate is higher that that at the smaller end plate. The resulting force difference (F_{g1} -F_{g2}) is multiplied by the Q of the resonant assembly.

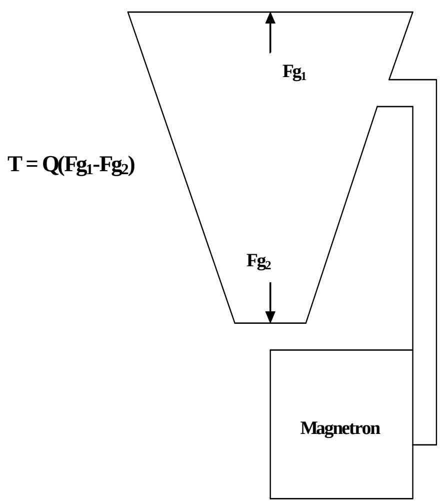

Fig 1

This force difference is supported by inspection of the classical Lorentz force equation (reference 1).

$$F = q(E + vB) \qquad (1)$$

If v is replaced with the group velocity v_g of the electromagnetic wave, then equation 1 illustrates that if v_{g1} is greater than v_{g2}, then F_{g1} should be expected to be greater than F_{g2}.

However as the velocities at each end of the waveguide are significant fractions of the speed of light, a derivation of the force difference equation invokes the difference in velocities and therefore must take account of the special theory of relativity.

Relativity theory implies that the electromagnetic wave and the waveguide assembly form an open system. Thus the force difference results in a thrust which acts on the waveguide assembly.

2. DERIVATION OF BASIC THRUST EQUATION

Consider a beam of photons incident upon a flat plate perpendicular to the beam. Let the beam have a cross-sectional area A and suppose that it consists of n photons per unit volume. Each photon has energy hf and travels with velocity c, where h is Planck's constant and f is the frequency. The power in the incident beam is then

$$P_0 = nhfAc \qquad (2)$$

The momentum of each photon is hf/c so that the rate of change of momentum of the beam at the plate (assuming total reflection) is $2nhfA$. Equating this change of momentum to the force F_0 exerted on the plate, we find

$$F_0 = 2nhfA = \frac{2P_0}{c} \qquad (3)$$

which is the classical result for the radiation pressure obtained by Maxwell (reference 2). The derivation given here is based on Cullen (reference 3). If the velocity of the beam is v then the rate of change of momentum at the plate is $2nhfA(v/c)$, so that the force F_g on the plate is in this case given by

$$F_g = \frac{2P_0}{c}(v/c) \qquad (4)$$

We now suppose that the beam enters a vacuum-filled waveguide. The waveguide tapers from free-space propagation, with wavelength λ_0, to dimensions that give a waveguide wavelength of λ_g and propagation velocity v_g. This is the group velocity and is given by

$$v_g = \frac{c}{\sqrt{\mu_r e_r}} \frac{\lambda_0}{\lambda_g} \qquad (5)$$

Then from (4) and (5) (with $\mu_r = e_r = 1$) the force on the plate closing the end of the waveguide is

$$F_g = \frac{2P_0}{c}(v_g / c) = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_g} \qquad (6)$$

see Cullen (p.102 Eq. (15)).

Assume that the beam is propagated in a vacuum-filled tapered waveguide with reflecting plates at each end. Let the guide wavelength at the end of the largest cross-section be λ_{g1} and that at the smallest cross-section be λ_{g2}. Then application of (6) to each plate yields the forces

$$F_{g1} = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_{g1}}, \quad F_{g2} = \frac{2P_0}{c}\frac{\lambda_0}{\lambda_{g2}}$$

Now $\lambda_{g2} > \lambda_{g1}$, due to the difference in cross-section, and hence $F_{g1} > F_{g2}$.
Therefore the resultant thrust T will be

$$T = F_{g1} - F_{g2} = \frac{2P_0}{c}\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}}\right) \qquad (7)$$

3. DERIVATION OF NET THRUST

From (7) it can be seen that to maximise thrust, the taper design should ensure λ_{g1} approaches λ_0 consistent with an acceptable maximum dimension. Also λ_{g2} should approach infinity, which occurs when the minimum dimension approaches the propagation cut-off limit. This minimum dimension must be consistent with allowable manufacturing and thermal tolerances.

The resulting design must also ensure a low taper slope, to minimise the axial component of side wall forces. This combination of dimensional constraints requires an iterative numerical design approach, taking account of the highly non-linear relationship between radial dimensions and guide wavelengths. This relationship is illustrated in fig 2.

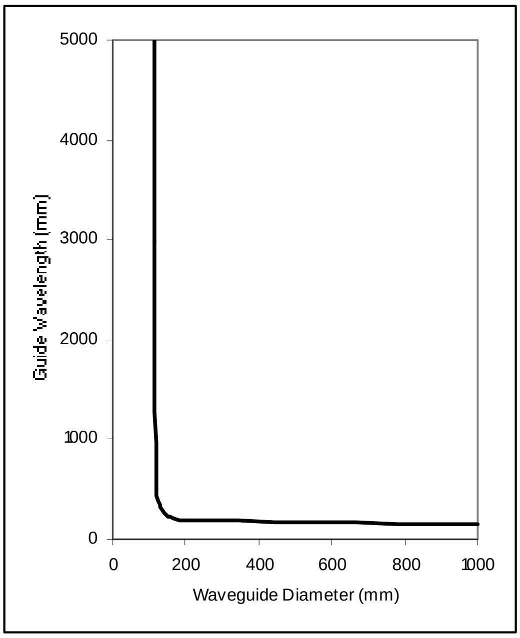

Fig 2 Guide wavelength for circular TMO1 at 2 GHz

It is clear that if the minimum dimension was the cut off diameter, force F_{g2} would be zero. However because there would still be a significant small end plate area, the projected area of the side wall would not equal the area of the large end plate. Thus any attempt to show a resultant zero net force due to equalisation of areas is incorrect.

Note also that if the forces had been the mechanical result of a working fluid within the closed waveguide assembly, then the resultant force would merely introduce a mechanical strain in the waveguide walls. This would be the result of a closed system of waveguide and working fluid.

In the present system, the working fluid is replaced by an electromagnetic wave propagating close to the speed of light and Newtonian mechanics must be replaced with the special theory of relativity. There are two effects to be considered in the application of the special theory of relativity to the waveguide. The first effect is that as the two forces F_{g1} and F_{g2} are dependent upon the velocities v_{g1} and v_{g2}, the thrust T should be calculated according to Einstein's law of addition of velocities given by

$$v = \frac{v_1 + v_2}{1 + (v_1 v_2)/c^2}$$

The second effect is that as the beam velocities are not directly dependent on any velocity of the waveguide, the beam and waveguide form an open system. Thus the reactions at the end plates are not constrained within a closed system of waveguide and beam, but are reactions between waveguide and beam, each operating within its own reference frame, in an open system.

From (7) and (5) we find

$$T = \frac{2P_0}{c}\left(\frac{v_{g1}}{c} - \frac{v_{g2}}{c}\right)$$

where

$$v_{g1} = c\lambda_0 / \lambda_{g1} \quad v_{g2} = c\lambda_0 / \lambda_{g2}.$$

Applying the above addition law of relativistic velocities we obtain

$$T=\frac{2P_0}{c^2}\left(\frac{v_{g1}-v_{g2}}{1-v_{g1}v_{g2}/c^2}\right)=\frac{2P_0S_0}{c}\left(\frac{\lambda_0}{\lambda_{g1}}-\frac{\lambda_0}{\lambda_{g2}}\right) \quad (8)$$

where the correction factor S_o is

$$S_0=\left\{1-\frac{\lambda_0^2}{\lambda_{g1}\lambda_{g2}}\right\}^{-1}$$

We suppose that the waveguide is resonant at the frequency of the microwave beam and that the conductive and dielectric losses are such that there are Q return paths (each at power P_0). Then the total thrust is finally given by

$$T=\frac{2P_0QS_0}{c}\left(\frac{\lambda_0}{\lambda_{g1}}-\frac{\lambda_0}{\lambda_{g2}}\right) \quad (9)$$

4. CONSERVATION OF MOMENTUM

The concept of the beam and waveguide as an open system can be illustrated by increasing the velocity of the waveguide in the direction of the thrust, until a significant fraction of the speed of light is reached. Let v_w be the velocity of the waveguide. Then as each plate is moving with velocity v_w the forces on the plates, given by equation 6, are modified as follows:

$$F_{g1}=\frac{2P_0}{c^2}\left(\frac{v_{g1}-v_w}{1-v_{g1}v_w/c^2}\right)=\frac{2P_0}{c^2}v_{ga}$$

and

$$F_{g2}=\frac{2P_0}{c^2}\left(\frac{v_{g2}+v_w}{1+v_{g2}v_w/c^2}\right)=\frac{2P_0}{c^2}v_{gb}$$

The thrust is then given by

$$T=\frac{2P_0}{c^2}\left(\frac{v_{ga}-v_{gb}}{1-v_{ga}v_{gb}/c^2}\right) \quad (10)$$

The solution to (10) is illustrated in Fig 3. Note that to maintain the principle of the conservation of momentum, the acceleration of the waveguide due to thrust, is opposite to the actual thrust direction. Thus, in Fig 3, the sign convention for the waveguide velocity axis is:

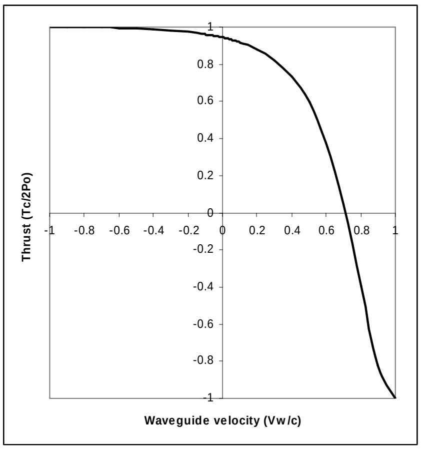

Fig 3. Solution to equation 10

When the waveguide is accelerated along the acceleration vector, the thrust approaches a maximum of 1. However, as the velocity of the waveguide increases in the direction of thrust, the thrust will decrease to zero. This point is reached when $v_{ga} = v_{gb}$. Fig 2 illustrates the solution to equation 9 for values of $v_{g1} = 0.95\,c$ and $v_{g2} = 0.05c$. It can be seen that if v_w is increased beyond the value of 0.7118c, the thrust reverses.

Equation 10 illustrates that the thruster is an open system, where guide velocities are independent of waveguide velocity, and it is the relative velocities that give rise to the forces. Note that if Einstein's law for the addition of velocities had not been used, relative velocities would exceed c, and the thrust would go above the theoretical limit of 1.

5. CONSERVATION OF ENERGY

We now examine the implications of the principle of the conservation of energy when the thrust is first measured on a static test rig, and then when an engine is used to accelerate a spacecraft.

With the microwave engine mounted on a static test rig, all the input power P_0 is converted to electrical loss. In this case the Q of the engine may be termed Q_u, the unloaded Q.

Now

$$Q_u = \frac{P_c}{P_0} = \frac{P_c}{P_e}$$

where P_c is the circulating power within the resonant waveguide and P_e is the electrical loss. From (9) we find

$$T = \frac{2P_0 D_f Q_u}{c},$$

Where D_f is the design factor

$$D_f = S_0\left(\frac{\lambda_0}{\lambda_{g1}} - \frac{\lambda_0}{\lambda_{g2}}\right)$$

Then $$T = \frac{2D_f P_c}{c}. \qquad (11)$$

Thus if the circulating power remains constant, for instance in a superconducting resonant waveguide, then T will remain constant. This will be important in non spacecraft applications where very high values of Q_u could be employed to provide a constant thrust to counter gravitational force.

If the engine is mounted in a spacecraft of total mass M and is allowed to accelerate from an initial velocity v_i to a final velocity v_f in time Δt, then by equating kinetic energies we obtain:

$$P_k \Delta t = \frac{M}{2}\left(v_f^{\,2} - v_i^{\,2}\right)$$

where P_k is the output power transferred to the spacecraft. From this we obtain

$$P_k \Delta t = \frac{M}{2}\left(v_f - v_i\right)\left(v_f + v_i\right),$$

so that $$P_k = M\bar{v}a \qquad (12)$$

where $\bar{v}$ is the average velocity over time Δt and a is the acceleration of the spacecraft.

Now $M.a$ is the force due to the acceleration of the spacecraft, which opposes the thrust of the engine. Then

$$P_k = \frac{2P_0 Q_l D_f \bar{v}}{c} \qquad (13)$$

where Q_l is the loaded Q of the engine when it is delivering an output power P_k.

The electrical power losses P_e are assumed to be I^2R losses and thus for any value of Q ,

$$P_e = Q^2 P_{e0}$$

where P_{e0} is the loss for $Q=1$. From the static case, we have

$$P_{e0} = \frac{P_0}{Q_u^{\,2}}$$

so that

$$P_e = P_0\left(\frac{Q_l}{Q_u}\right)^2 \qquad (14)$$

For an accelerating spacecraft,

$$P_0 = P_e + P_k$$

Substitution of (13) and (14) into this last equation then yields

$$\left(\frac{Q_l}{Q_u}\right)^2 + \frac{2Q_l D_f \bar{v}}{c} = 1 \qquad (15)$$

Fig 4 shows the solution to (15) for values of $\bar{v}$ up to 10km/sec and for values of Q_u equal to $5\text{x}10^3$, $5\text{x}10^4$ and $5\text{x}10^5$. The value of D_f is taken to be 0.945.

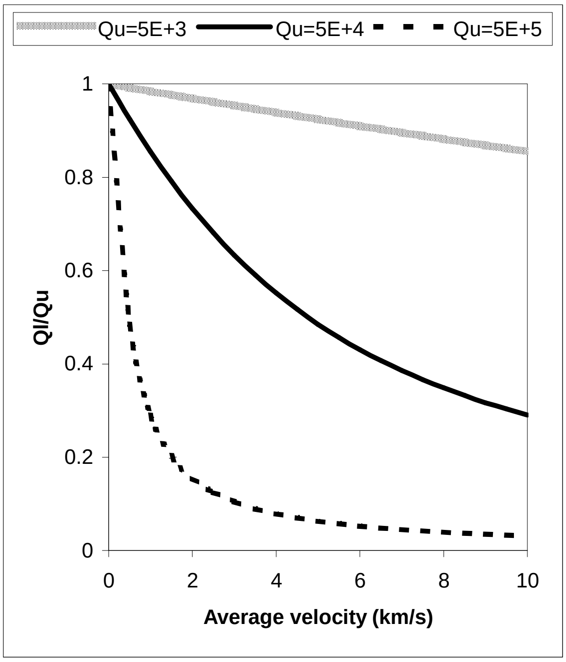

Fig 4 Solution to equation 15.

For D_f equal to 0.945 and an average velocity of 3 km/s, the specific thrust is obtained from (9) and (15) and is given in fig 5. This illustrates that the specific thrust increases to a maximum of 333 mN/kW at this velocity.

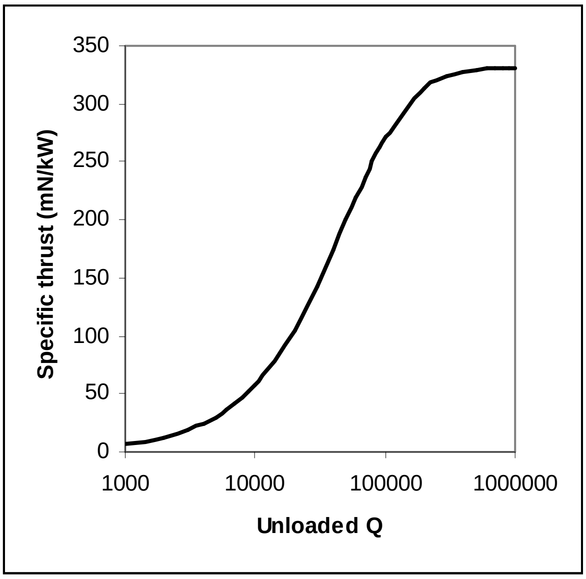

Fig 5 Specific thrust at 3km/s.

6. EXPERIMENTAL THRUSTER

A 160 mm diameter experimental thruster, operating at 2.45 GHz was designed and built. (see fig 6) The design factor, calculated from as-built measurements of the thruster geometry was 0.497. An unloaded Q of 5,900 was measured. The maximum thrust, measured using a precision balance was 16mN for an input power of 850W, which is very close to the thrust of 16.6mN predicted from equation (9).

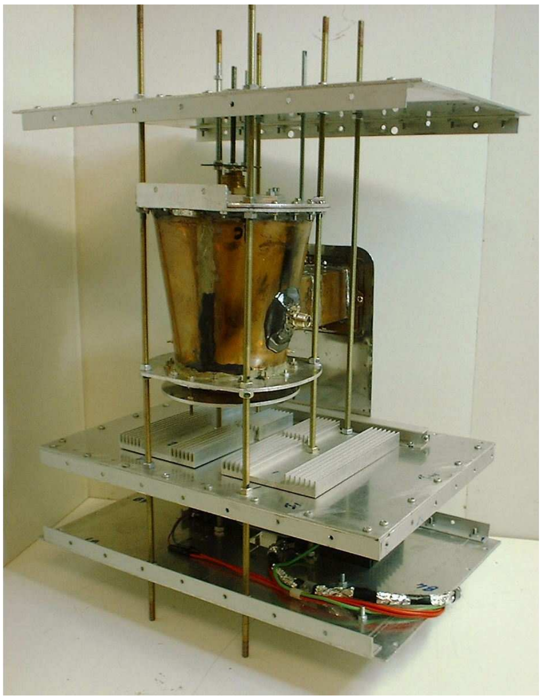

Fig 6 Experimental Thruster

The thrust could be varied from zero to maximum by varying the input power, or by varying the resonant frequency of the thruster. Considerable efforts were made to test for possible thermal and electromagnetic spurious effects. The primary method was to carry out all tests in both nominal and inverted orientations, and to take the mean of the results. The thruster was also sealed into a hermetic enclosure to eliminate buoyancy effects of the cooling air. Three different types of test rig were used, two using 1 mg resolution balances in a counterbalance test rig and one using a 100 mg resolution balance in a direct measurement of thruster weight.

Comparison of the rates of increase of thrust for the different spring constants, using pulsed input power, gave a clear proof that the thrust was produced by momentum transfer and was not due to any "undefined" spurious effect.

The total test programme encompassed 450 test runs of periods up to 50 seconds, using 5 different magnetrons.

7. DEMONSTRATOR ENGINE

Unlike the experimental thruster, the Demonstrator Engine was rated for continuous operation and extensive design work was required to increase the specific thrust by raising the design factor and unloaded Q.

The engine was built to operate at 2.45 GHz, with a design factor of 0.844 and has a measured Q of 45,000 for an overall diameter of 280 mm. The microwave source is a water cooled magnetron with a variable output power up to a maximum of 1.2 kW.

To obtain the predicted thrust, the engine was required to maintain stable resonance at this high Q value. Major design challenges included thermal compensation, tuning control and source matching.

The engine was tested in a large static test rig employing a calibrated composite balance to measure thrust in both vertical and horizontal directions. A total of 134 test runs were carried out over the full performance envelope.

Fig 7 gives test results for 3 Vertical Thrust test runs under the same input and tuner conditions but for thrust vectors in the Up, Down and Horizontal directions. This clearly illustrates the loss of measured weight for the Up vector, the increase in measured weight for the Down vector, and a mean weight change close to zero, for the horizontal vector. These early comparative tests yielded specific thrusts around 80mN/kW.

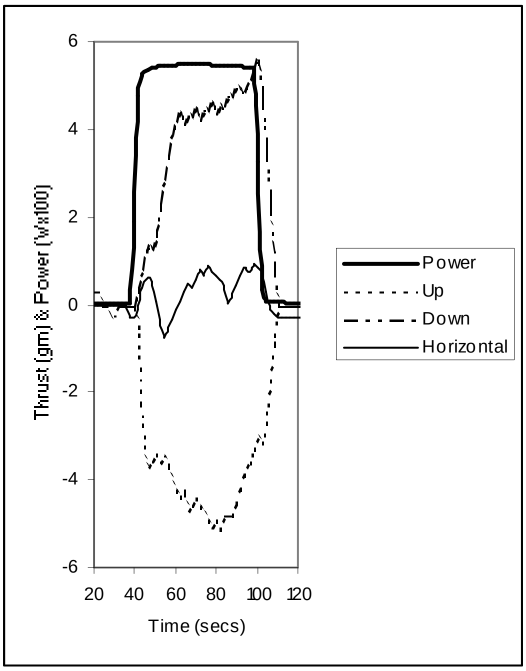

Fig 7. Demonstrator Engine Static Test Data

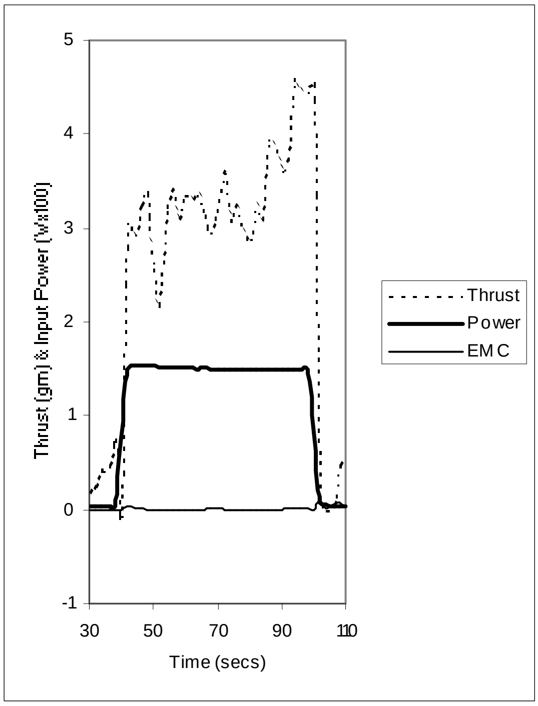

Fig 8. Electromagnetic Compatibility (EMC)

Fig 8 shows the results for a test run with the engine on the balance and then with it suspended above the balance. This illustrates the thrust measurements were not subject to EMC effects. Specific thrust for this test was 214mN/kW.

The engine was then mounted on a dynamic test rig enabling it to be "flown" on a rotary air bearing, as shown in fig 9.

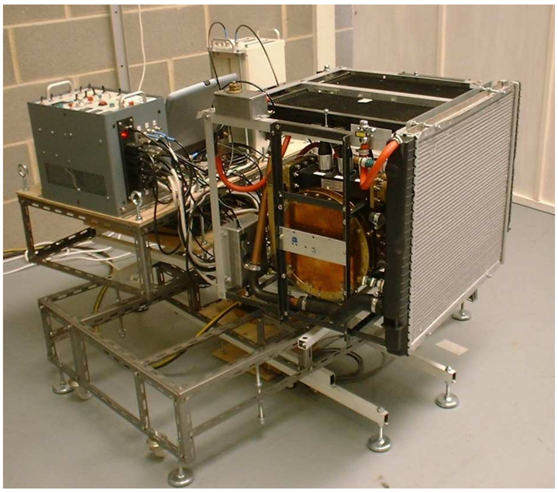

Fig 9. Demonstrator Engine on Dynamic Test

The tests simulated the engine moving a 100Kg spacecraft in weightless conditions. The test programme included acceleration and deceleration runs in both directions, and confirmed the thrust levels measured in the static tests.

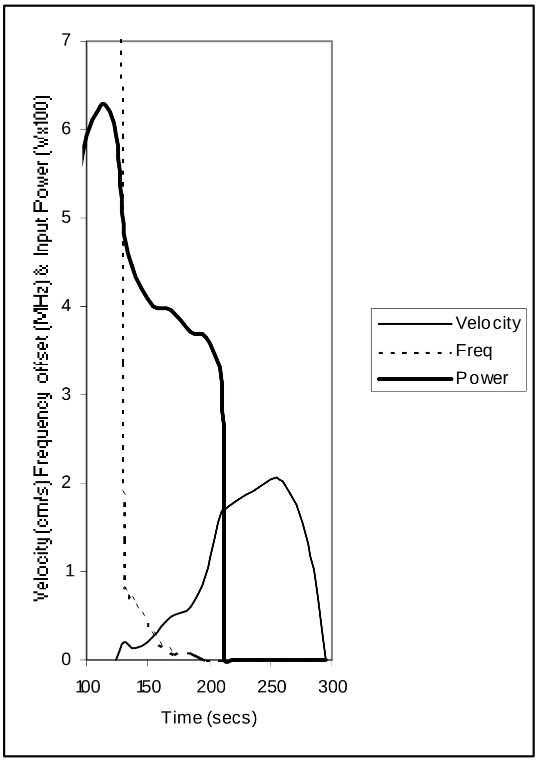

Fig 10. Dynamic test results

Fig 10 gives the result of a typical test run, where the Demonstrator Engine produced a thrust of 10.4 gm against a calibrated friction torque of 7.1 gm. Input power was 421W, giving a specific thrust of 243 mN/kW.

The frequency offset curve shows that initial magnetron thermal drift ends with frequency lock. At this point, 130 secs into the test run, the velocity data shows the start of acceleration under power. The prior thermal drift period, with no acceleration, shows that the thrust is not a result of spurious thermal effects. When the power is turned off, at 210 secs, there is a coast period as the slosh effects of 5kg of coolant maintain a reduced acceleration. This is followed by the deceleration due to the friction torque. A maximum velocity of 2cm/s was achieved and a total distance of 185cm was "flown".

The direction of acceleration was opposite to the direction of thrust, thus conclusively proving that the engine obeys Newton's laws, and that although no reaction mass is ejected, the engine is not a reactionless machine. An electrical reaction occurs between the EM wave and the reflector surfaces of the resonator, resulting in an input impedance change with acceleration. This is seen in the power curve in fig 10.

8. CURRENT PROGRAMMES

Current programmes include an experimental superconducting thruster. This low power, HTS device operates at liquid nitrogen temperature, and is designed for very high Q and consequently high specific thrust.

The thruster operates at 3.8 GHz, and was designed using an update of the software used for the previous S band designs. Superconducting surfaces are formed from YBCO thin films on sapphire substrates.

Small signal testing at –195 deg C has confirmed the design, with a Q of 6.8×10^6 being measured.

A second programme covers the design and development of a 300 Watt C Band flight thruster. This has a specified thrust of 85 mN, and a mass of 2.92Kg. Overall dimensions are

265mm diameter at the baseplate and a height of 164mm.

The initial design of the flight thruster is complete, and a specification has been issued. Phase 1 of the thruster development has started.

9. DEMONSTRATOR SATELLITE

A Demonstrator Satellite proposal has also been prepared, based on an existing 100 kg microsatellite design, propelled by the flight thruster.

Following launch to LEO, the continuous manoeuvrability important for military and formation flying missions would be demonstrated. This would be followed by a spiral transfer from LEO to GEO to illustrate the large financial savings possible in communication or solar power satellite launches. Finally the spacecraft would escape Earth's gravity and, after a total of 7 years continuous operation, reach 16.5 km/sec. Depending on the flight path chosen, a modest science mission could be incorporated in this cruise phase.

The spacecraft would be based on a standard DMC series microsatellite. The imaging payload and propulsion system would be replaced by a C Band EmDrive engine as described in Section 8. A small science payload (<5 kg) could be carried. The total spacecraft mass would be less than 90 kg.

Assuming 100W of DC power is available to the EmDrive engine, a static thrust of 19mN would be achieved. The thrust would be continuously variable from zero to maximum by control of the input power. At the terminal velocity of 16.5 km/sec maximum thrust would be reduced to 3.5mN. This is the effect of equation (15) and monitoring the progress of the cruise phase will give confirmation of this equation.

The EmDrive engine would be mounted in the payload section with the thrust vector aligned with the central axis of the spacecraft. The engine would consist of a single thruster, a fully redundant Frequency Generating Unit and two Travelling Wave Tube Amplifiers. These units, together with isolators, combiner and cables would be mounted on a thermally radiating baseplate designed for simple integration with the spacecraft bus. All power, telemetry and command interfaces would be designed for direct compatibility with the standard spacecraft subsystems.

10. CONCLUSIONS

A theory of EmDrive operation has been developed with the static and dynamic thrust equations derived from basic electromagnetic theory. Following extensive review, no contravention of the laws of the conservation of momentum and conservation of energy has been identified. Design software, based on the thrust equations, was developed to enable an experimental thruster to be manufactured. This thruster was subjected to a comprehensive test programme, which proved the concept and verified the design software.

A Demonstrator Engine, with improved design and higher Q, was then tested in both static and dynamic test rigs. The results showed a significant increase in thrust, consistent with the design predictions. Correct dynamic operation was demonstrated, consistent with Newton's laws.

With the basic research on first generation EmDrive completed successfully, work has moved into the flight development phase. Further research into second generation, superconducting technology has started, with initial test data giving the predicted large increase in Q.

A microsatellite proposal illustrates that a very modest flight project could demonstrate the major advantages to be gained in commercial, scientific and military missions, by employing EmDrive technology.

ACKNOWLEDGEMENTS

The author is grateful for the assistance given by colleagues in SPR Ltd, by Dr R B Paris of Abertay University, Dundee, by J W Spiller of Astrium UK Ltd and by Professor J Lucas of The University of Liverpool. The early theoretical work and experimental programmes were carried out with support from the Department of Trade and Industry under their SMART award scheme, and then under a Research and Development grant.

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BIOGRAPHY

The author is a Chartered Engineer and Director of SPR Ltd. Early career experience included Research and Development on small gas turbines, guided missiles, radars, and communication systems. This was followed by 20 years in the Space industry at EADS Astrium, which included appointments as Head of Department for Payload Equipment, and Project Manager for a number of large communications payloads. Responsibilities also covered the initial design of the Galileo navigation payload and signal structure.