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A RESONANT-CAVITY TORQUE-OPERATED WATTMETER FOR MICROWAVE POWER

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SUMMARY

A sensitive method of microwave power measurement is described which makes use of the mechanical force exerted by the electromagnetic field on a small vane in a resonant cavity. It is shown that the force on the vane is a simple function of the Q-factor of the cavity, the power absorbed in it and the perturbation of its resonant frequency caused by the vane. The results of a comparison between an experimental wattmeter based on this principle and a water calorimeter are given, and the requirements of a practical instrument are discussed.

(1) INTRODUCTION

Most of the devices commonly used for accurate power measurement at microwave frequencies absorb the power in a lossy material and measure the resultant temperature rise. Calibration is carried out by the application of power from d.c. sources, and the assumption is made that the temperature rise is the same for equal d.c. and microwave powers. The thermistor, bolometer, thermocouple, and water calorimeter are all used in this way; none of them gives an absolute measurement of power, although the calorimeter is generally regarded as a standard.

More recently the effect of radiation pressure at microwave frequencies has been demonstrated by Carrara and Lombardini,[1] and used by Cullen[2,3,4] to measure the power in a waveguide. In its later form Cullen's instrument consists of a rectangular guide containing a flat metal vane suspended so that it can rotate about an axis parallel to the broad dimension of the guide. The electric field of the H_{01} mode exerts a force on the vane which tends to rotate it to the transverse position. The torque on the vane is proportional to the power, and the constant of proportionality may be found from a subsidiary experiment using a movable piston and standing-wave indicator. The calibration is effected by means of measurements of mass, length and time only, and hence the instrument makes an absolute measurement of power. It is used as a transmission wattmeter and absorbs negligible power from the guide. With this type of suspension the vane acts, roughly, as a voltmeter across the guide and hence for accurate power measurement the standing-wave ratio of the load must be nearly unity. The magnitude of the torque is a function of the electric field strength and the vane size. In Cullen's instrument it is of the order of 10^{-4} dyne-cm/watt which is rather small for measurements other than laboratory ones at powers less than 10 watts.

Evidently, if the power to be measured is fed into a cavity resonator of high Q-factor containing a vane, the field at the vane (and hence the deflection sensitivity) may be made large. Further, it can be shown that the power absorbed in the cavity may be calculated from the force acting on the vane without any direct knowledge of the field distribution in its vicinity.

An instrument based on this principle has been made and its design is described.

Correspondence on Monographs is invited for consideration with a view to publication.

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(2) THEORY

The basis of the calculation for the force acting on the vane is a theorem of adiabatic invariance expressed by Maclean,[5] which states that in a lossless electromagnetic resonator the action of each mode, i.e. the product of total energy and period, is invariant against an adiabatic deformation.

Consider a lossless system consisting of a cavity, resonant in one mode with period τ, containing a small vane and having stored energy W. In general, a force will be exerted on the vane. Let it move slowly a small distance ds. This movement will alter the resonant period of the cavity, but, by the theorem, we have

$$W\tau = \text{a constant}$$

from which

$$Wd\tau + \tau dW = 0$$

and

$$dW = -\frac{W}{\tau}d\tau$$

Since the system is lossless, the change in energy stored by the field must be equal to the work done by the moving vane. Therefore

$$Fds = -dW = W\frac{d\tau}{\tau}$$

$$F = \frac{W}{\tau}\frac{d\tau}{ds}$$

where F is the force acting in the direction ds.

In a practical system the resonator will not be lossless, but the field distribution in a resonator of high Q-factor will be negligibly different from that in a lossless one, and for a given stored energy the force on the vane will be the same.

In a lossy resonator

$$Q = \frac{2\pi \times \text{energy stored}}{\text{Energy lost per cycle}}$$

$$= \frac{2\pi W}{P\tau}$$

where P is the power fed into the cavity. Therefore

$$W = \frac{QP\tau}{2\pi}$$

and

$$F = \frac{QP}{2\pi}\frac{d\tau}{ds}$$

A better form for this expression is obtained if Q is replaced by $\pi f t_0$, where t_0 is the time-constant of decay of oscillation amplitude in the cavity, and f is the frequency. Then

$$F = \frac{\pi f t_0 P}{2\pi}\frac{d\tau}{ds}$$

$$= P\frac{t_0}{2}\frac{1}{\tau}\frac{d\tau}{ds} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$= -P\frac{t_0}{2}\frac{1}{f}\frac{df}{ds} \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

For a rotating vane $T = -P\frac{t_0}{2}\frac{1}{f}\frac{df}{d\theta}$

where T is the torque on the vane.

For microwave cavities of similar shape and mode, Q/λ is inversely proportional to the skin depth δ. Now

$$\delta = \sqrt{\frac{\rho}{\pi\mu f}}$$

where ρ is the resistivity of the cavity wall. Therefore

$$t_0 = \frac{Q}{\pi f} \text{ and } \propto \frac{1}{(f)^{3/2}}$$

Therefore the sensitivity of the device as a power meter is inversely proportional to (frequency)$^{3/2}$ for a constant fractional rate of detuning, $\frac{1}{f}\frac{df}{d\theta}$.

(3) SOME PRACTICAL CONSIDERATIONS

Slater[6] has shown that if a cavity is perturbed, by pushing in its boundary, the change of resonant frequency is dependent on the integral $\int (H^2 - E^2)dv$ over the volume which is removed from the cavity by the perturbation of the wall. The resonant frequency decreases if the perturbation is made at a point of predominant electric field, and increases if the perturbation is made at a point of predominant magnetic field. At some intermediate point it is possible for the effects to cancel. Similarly, for a simple small vane, df_0/ds, and hence the force, is greatest if the vane is placed in a region where one field is predominant and if the vane is of such a shape that it couples more strongly to that field than to the other.

A cylindrical H_{011} resonator was used to test the theory expressed above. At the mid-section along its length the E field in this cavity is circumferential, with a maximum at $0{\cdot}48r$, where r is the radius of the cavity. The H field is axial, having maxima at the axis and the wall and a zero at $0{\cdot}62r$. Vanes were mounted on radial polystyrene rods and the resonant frequency and Q-factor of the cavity were measured as the rods were rotated.

A short, straight copper rod mounted parallel to the axis and placed in the strong E field at about half the radius caused a decrease of resonant frequency as it was rotated from the axial to the transverse position. The change of frequency with rotation roughly followed a $\sin^2\theta$ law. The Q-factor of the cavity decreased as the vane was rotated; hence the force on the vane, which is proportional to the product of Q and $df_0/d\theta$, reached a maximum when the vane made an angle of about 38° with the axis. In Fig. 1, typical curves of Q, Δf_0, and $Qdf_0/d\theta$ are shown.

A wire loop or a disc, suspended so that its centre was on the cavity axis, caused an increase of resonant frequency as its plane was rotated from an axial to a transverse position. By displacing the centre of a vane of this type off the axis it was possible to find a point where its coupling to the magnetic and electric fields was such that its rotation caused very little change in the resonant frequency.

For a given value of $df_0/d\theta$, a straight rod caused less depression of Q than a loop; it was therefore decided to use a rod in the experimental instrument built. The instrument was checked against a water calorimeter.

(4) CONSTRUCTION

A drawing of the cavity and vane system is given in Fig. 2. The cavity used was an S-band echo box, 6 in in diameter and about 4 in long, containing a raised ring round the circumference of the base to remove the H_{01}–E_{11} degeneracy. The vane, a silver-plated rod $0{\cdot}095$ in in diameter and $1{\cdot}275$ in long, was suspended $1{\cdot}5$ in from the axis of the box by a $0{\cdot}06$ in-diameter polystyrene rod. This rod was attached, outside the cavity wall, to a stiff wire shaft carrying a small mirror and a metal foil spider which dipped into an annular pool of liquid for damping the motion of the vane. The suspension was a 6 in length of No. 49 s.w.g. phosphor-bronze wire attached to a torsion head at its upper end.

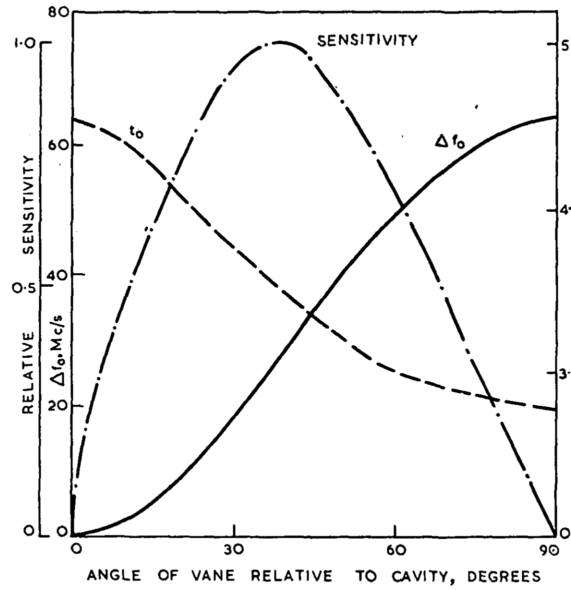

Fig. 1.—Typical curves of t_0, Δf_0, and sensitivity $\left(t_0\frac{df_0}{d\theta}\right)$ for a rod in an H_{011} cavity.

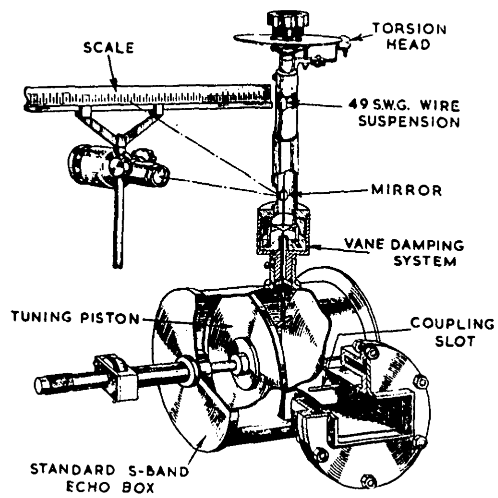

Fig. 2.—Experimental wattmeter.

(5) CALIBRATION

The torsional constant of the suspension was found by observing the period of rotational oscillation of a $\frac{1}{4}$ in steel ball attached to its lower end.

In order to find the value of $df_0/d\theta$ for the vane and cavity over the working range of deflection, the vane was rotated manually by the torsion head and the resonant frequency of the cavity for a series of deflections was measured, using a sufficiently small input to cause negligible force on the vane.

The Q-factor of the cavity was measured by the R.R.D.E. echo-box Q-factor meter,[7] which equates the rate of decay of free oscillation in the cavity with the decay of voltage across a known parallel *RC* circuit. This part of the calibration is therefore not absolute. However, the Q-factor could have been found in terms of the change of cavity impedance with applied frequency and hence measured by a piston and standing-wave-indicator experiment involving measurement of length and time only. It would have been difficult to equal the accuracy of the Q-meter by this method because of the high Q-factor of the resonator.

(6) OPERATION

When making a measurement, power is fed to the cavity through a slot coupling whose length is adjusted so that the resonant cavity appears as a matched load to the guide. The piston is then used to tune the cavity to resonance. When the field inside builds up, the vane moves, thereby lowering the resonant frequency of the cavity. In this condition the vane is in a stable state. If it returns towards its zero position, the force acting on it increases as it brings the box nearer to resonance; its motion in the other direction is restrained by the suspension. Further tuning is required to obtain a greater deflection.

In the final position the torque in the suspension is equal to the maximum torque that can be exerted by the field with the cavity fully at resonance. At this point the vane becomes unstable; movement of the vane towards its zero position detunes the cavity, thus reducing the deflecting torque, and the vane swings back to zero.

A measurement therefore consists in tuning the cavity slowly to obtain the maximum vane deflection and calculating the power from the values of torque, Q and $df_0/d\theta$ at that vane position.

(7) EXPERIMENTAL RESULT

(7.1) Measurements

A sketch of the arrangement used to check the experimental accuracy of the cavity wattmeter is given in Fig. 3.

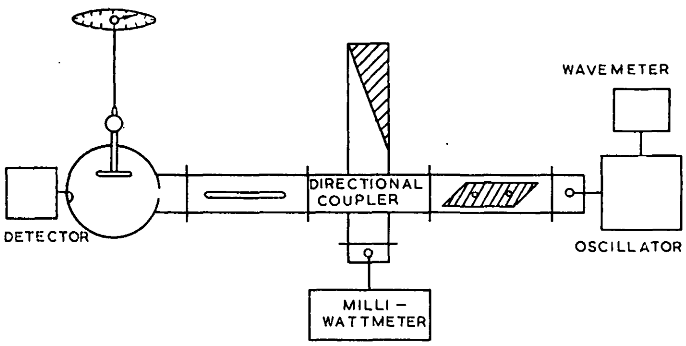

Fig. 3.—Apparatus used for testing the experimental cavity wattmeter.

The power absorbed in the cavity was measured by means of a directional coupler of known power division and a thermistor milliwattmeter which had been calibrated against a water calorimeter. Simultaneous measurements of the power indicated by the milliwattmeter and by the cavity wattmeter were made at various power levels.

In Table 1 the results of eleven such measurements are given.

Table 1

POWER ABSORBED; OBSERVED AND CALCULATED

P_w	D	T	t_0	P_c
mW	cm	dyne-cm $\times 10^{-3}$	µs	mW
24·2	5·4	8·84	3·52	22·04
24·2	5·8	9·50	3·52	23·75
25·6	6·1	10·00	3·52	24·85
28·35	7·4	12·12	3·50	30·4
34·59	8·2	13·4	3·50	33·6
42·4	10·5	17·2	3·48	43·4
48·4	11·4	18·7	3·48	47·1
49·7	11·4	18·7	3·48	47·1
54·6	12·34	20·2	3·46	51·25
63·6	14·8	24·25	3·42	62·3
67·5	15·4	25·22	3·42	64·7

Where P_w = Power absorbed in the cavity as indicated by the milliwattmeter and coupler.
D = Deflection at 67 cm radius.
T = Torque.
t_0 = Unloaded time-constant of the cavity.
P_c = Power absorbed in the cavity, calculated from the formula

$$P_c = \frac{2T}{t_0}\frac{f}{df/d\theta} \times 10^{-1} \quad . \quad . \quad . \quad . \quad (2)$$

The denominator $df/d\theta$ was assumed constant over the range of deflections used and was equal to 63·8 Mc/s per radian.

The sensitivity of the instrument in this experiment was about 0·4 dyne-cm/watt. The values of P_w and P_c are plotted in Fig. 4, where it may be seen that the cavity wattmeter reads true power within a few per cent.

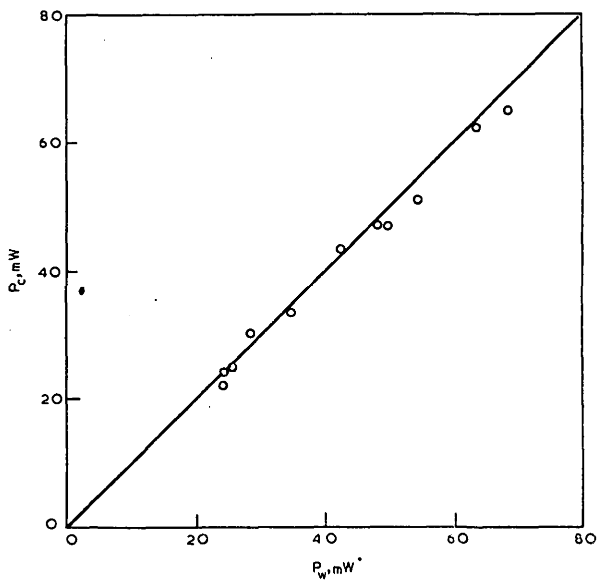

Fig. 4.—Power indication of the cavity wattmeter (P_c) plotted against power indicated by the calibrated milliwattmeter (P_w).

(7.2) Errors

The possible systematic errors amounted to about $7\frac{1}{2}\%$. The sum of the errors in the calibration of the milliwattmeter and the directional coupler was about $3\frac{1}{2}\%$, while the measurement of the constants of the cavity wattmeter, t_0, $df_0/d\theta$ and the specific torque of the suspension, added about 4%.

The large random errors were chiefly due to the difficulty of tuning the cavity without upsetting the delicate equilibrium of the vane.

(8) FUTURE DEVELOPMENT

Although the power meter has been tested at 3 000 Mc/s only, the principle may be applied at any other frequency, and in practice its main use may be at Q-band or at frequencies of the order of a few hundred megacycles per second where convenient power standards are lacking.

There are a number of difficulties in constructing a practical instrument on the resonant-circuit principle. In the form described in the paper, for instance, the instability in the vane deflection makes the tuning of the cavity, to obtain maximum deflection, very tedious. Furthermore, the bandwidth of the cavity is small because of the high Q-factor, and the device has the undesirable property of presenting a rapidly varying reactive load to the power source.

The cause of the instability of the vane is analysed in Section 11.2, where it is shown that the instability exists only if the vane deflection exceeds certain limits. It is proposed to increase the effective suspension stiffness and to measure the deflecting torque electrically by means of a servo mechanism attached to the vane. The necessary increase in stiffness is of the order of 100 times. With this refinement the cavity could be tuned rapidly to obtain the maximum torque on the vane, and, because of the very small vane movement, changes in Q and $df_0/d\theta$ with vane rotation would be insignificant.

The sensitivity of the instrument is proportional to the product of Q and $df_0/d\theta$, and in a practical wattmeter it would be preferable to work with a low Q-factor and a high value of $df_0/d\theta$ to obtain a larger operating bandwidth. For a loaded Q-factor of 1 000 at 3 000 Mc/s, the force on the vane is within 1% of its maximum value over a 300 kc/s band, which is adequate for most applications.

This reduction in Q-factor will also reduce the rate at which the cavity reactance changes with frequency, but to measure the power output of any source requiring a matched load an attenuating pad between the generator and cavity would be necessary.

(9) CONCLUSION

A sensitive method of power measurement at microwave frequencies has been demonstrated in which calibration does not involve comparison with a d.c. measurement but is derived from simple measurements of

(*a*) The Q-factor of a cavity.

(*b*) The torque on a vane in the cavity.

(*c*) The rate of change of resonant frequency of the cavity with rotation of the vane.

(10) ACKNOWLEDGMENTS

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(12) APPENDICES

(12.1) Derivation of Eqn. (1a), in a Simple Case

Eqn. (1*a*) for the force on a variable reactive element in a resonant system can be derived easily for a simple LCR circuit.

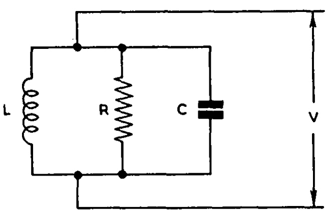

Fig. 5.—Simple LCR circuit.

In Fig. 5, let C be an idealized capacitor, so that

$$C = \frac{\epsilon_0 A}{x}$$

where A is the plate area and x is the plate spacing.

Then
$$f_0 \propto \frac{1}{\sqrt{C}}$$
$$= K'\sqrt{x} \quad \text{where } K' \text{ is a constant.}$$

Hence
$$\frac{df_0}{dx} = \frac{K'}{2\sqrt{x}}$$
$$= \frac{f_0}{2x} \quad \quad (3)$$

Also
$$t_0 = \frac{Q}{\pi f} = \frac{\omega CR}{\pi f}$$

from which
$$\frac{t_0}{2} = CR \quad \quad (4)$$

The power absorbed in the circuit is

$$P = \frac{V^2}{R} \quad \quad (5)$$

Again assuming an idealized capacitor with no fringing field, the force on the plate is

$$F = \frac{\epsilon_0}{2}E^2 A$$

where E is the electric field strength.

The mean force on the plates is therefore

$$F = \frac{\epsilon_0}{2}\frac{\overline{V^2}}{x^2}A$$

$$= \frac{C}{2}\frac{\overline{V^2}}{x}$$

$$= \frac{\overline{V^2}}{R}CR\frac{1}{f_0}\frac{f_0}{2x} \text{ in the direction of decreasing } x,$$

$$= -P\frac{t_0}{2}\frac{1}{f_0}\frac{df_0}{dx} \text{ from eqns. (3), (4), and (5).}$$

(12.2) Limits of Vane Deflection for Stability

It has been mentioned in Section 6 that the vane deflection passes through a region of instability as the cavity is tuned through resonance. Since this would be undesirable in a practical power meter, it is desirable to find the limits of vane deflection within which no instability occurs.

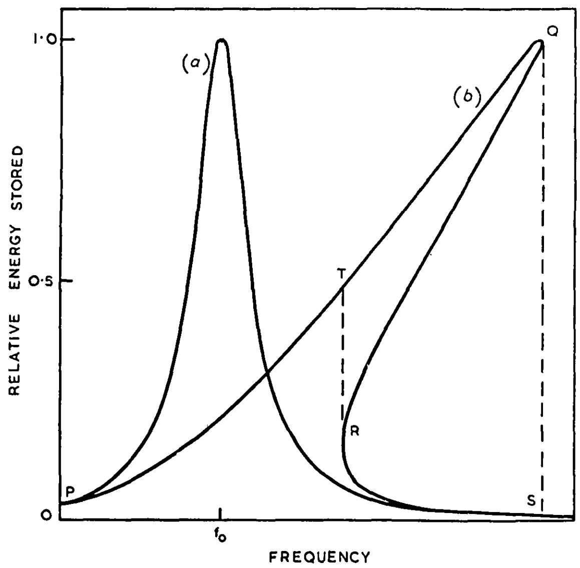

Fig. 6.—Normal and distorted frequency responses of a cavity containing a movable element.

In Fig. 6, curve (*a*) is the normal resonance curve of the cavity, with the vane fixed at its zero position, showing stored energy W plotted against frequency f. If the vane is free to move, however, it will be deflected by an angle proportional to W, and since, over a small range, the change of resonant frequency is proportional to the angle of deflection, each point on curve (*a*) will be displaced along the frequency axis by an amount proportional to W. The resultant effective resonance curve of the cavity and vane is then given by curve (*b*).

Over the region P–Q on this curve the vane is in stable equilibrium, since movement towards its zero increases the deflecting force by bringing the cavity nearer to resonance, and movement in the other direction increases the restoring force of the suspension. Over the region R–Q, however, it is unstable because the deflecting torque increases with vane rotation at a greater rate than the suspension torque, and if an attempt is made to trace the curve from S towards R, the vane becomes unstable in the vicinity of R and jumps to a point such as T. Similarly, near Q the vane will swing suddenly to point S.

At all stable points on the curve the torque in the suspension, T_s, is equal to the torque exerted by the field, T_f. Instability will occur if the angular rate of change of T_f is greater than the angular rate of change of T_s. For vane stability we require

$$\frac{dT_f}{T_f} < \frac{dT_s}{T_s}$$

i.e.

$$\frac{dW}{W} < \frac{d\theta}{\theta}$$

$$\frac{1}{W}\frac{dW}{df_0}\frac{df_0}{d\theta} < \frac{1}{\theta}$$

or

$$\theta\frac{df_0}{d\theta} < \frac{1}{\dfrac{1}{W}\dfrac{dW}{df_0}}$$

The frequency-dependence of stored energy, W, is of the form

$$W = \frac{1}{1+x^2} \quad \text{where } x = Q\left(1 + \frac{f^2}{f_0^2}\right)$$

$$\frac{1}{W}\frac{dW}{df_0} = \frac{-2x}{1+x^2}\frac{dx}{df_0}$$

$$= \frac{2x}{1+x^2}\frac{2Q}{f_0}\frac{f^2}{f_0^2}$$

$$\simeq \frac{2x}{1+x^2}\frac{2Q}{f_0}$$

over the region in which we are interested. The expression on the right is easily shown to have a maximum value when $x = 1$.

Then

$$\frac{1}{W}\frac{dW}{df_0} = \frac{2Q}{f_0}$$

The requirement for a stable system is

$$\theta\frac{df_0}{d\theta} < \frac{f_0}{2Q}$$

i.e. the maximum frequency deviation from f_0 due to vane movement must be less than half the bandwidth of the resonant circuit.

When the cavity is fed from a finite source-impedance, the relative bandwidth is the loaded bandwidth of the cavity, and in the particular case of a matched cavity it is $2f_0/Q$.

In the H_{01} cavity, Q (unloaded) $\simeq$ 30 000, and therefore the loaded bandwidth was about 200 kc/s. The maximum deflection of the vane for a power input of 50 mW was 0·085 rad.

Therefore

$$\theta\frac{df_0}{d\theta} = 0{\cdot}085 \times 63{\cdot}8\,\text{Mc/s}$$

$$= 5\,420\,\text{kc/s}.$$

An increase of over 50 times in the vane suspension stiffness would have been required to make the vane stable.

In the case of the variable capacitor in the lumped circuit of Section 11.1, the stability condition may be shown to be $\Delta x < (1/Q)x$, where Δx is the maximum plate movement, and x is the plate spacing.